Synthesis of Equivalent Circuit of Wireless Power Transfer Device Using Homogenization-based FEM

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In this work, the multi-turn coil used in a wireless power transfer (WPT) device is modeled as a uniform material using the homogenization method to consider the proximity effect. The frequency characteristics of the multi-turn coil can effectively be analyzed using the homogenization-based FEM. By fitting the coil impedance to the numerical results, the equivalent circuit of the multi-turn coil is synthesized for design and optimization of the power circuits in the WPT device. It is shown that the results obtained from the equivalent circuit and measurement are in good agreement.

***Index Terms*—Equivalent circuit, Homogenization, Skin and Proximity effects, Wireless power transfer.**

I. INTRODUCTION

WIRELESS POWER TRANSFER (WPT) has attracted great attentions for application in electric vehicles, biomedical devices and so on [1]-[2]. In order to downsize the coil for the WPT system, the driving frequency is increased. This results in significant changes in the coil impedance due to the skin and proximity effects. To accurately evaluate the coil impedance using conventional FEM, the coil must be subdivided so that the elements are smaller than the skin depth. The FE analysis, therefore, needs long computational time. To overcome this problem, the authors have proposed the homogenization method for the FE analysis in which the multi-turn coil is modeled as a uniform material with complex permeability in frequency domain [3]. When we analyze the WPT system including the multi-turn coil and power circuits with nonlinearity, time-domain FE analysis is required. The inverse Laplace transform based on the convolution would not be suitable for the time-domain analysis because of the long computational time.

In this paper, for the time-domain analysis of the WPT devices considering the skin and proximity effects, we synthesize the equivalent circuit of the WPT device using the homogenization-based FEM. In this method, the equivalent circuit is synthesized by curve fitting to the frequency responses [4]-[5] computed by the FEM. A WPT device including a rectifier is analyzed by the proposed method and the conventional method, and the numerical results are compared with the measured data.

II. FORMULATION

A. Homogenization-based FEM

The proximity effect can effectively be treated by introducing the complex permeability $\dot{\mu}_{\mathrm{r}}$. When we consider a round wire immersed in time-harmonic uniform magnetic field, $\dot{\mu}_{\mathrm{r}}$ is given by [3]

$$\dot{\mu}_{\mathrm{r}} = \mu_{\mathrm{r}} \frac{J_1(z)}{zJ_0(z) - J_1(z)} \tag{1a}$$

$$z = a(1-\mathrm{j})/\delta \tag{1b}$$

where μ_{r}, J_0, J_1, a and δ denote the relative permeability of the round wire, zeroth and first order Bessel functions, radius of the round wire and skin depth, respectively. The macroscopic complex permeability homogenized over the coil region can be obtained from the extended Ollendorff formula [3, 6]

$$\langle \dot{\mu} \rangle = \mu_0 \left\{ 1 + \frac{2\eta(\dot{\mu}_{\mathrm{r}} - 1)}{2 + (1-\eta)(\dot{\mu}_{\mathrm{r}} - 1)} \right\} \tag{2}$$

where μ_0 and η denote the permeability of vacuum and the volume fraction, respectively. The coil permeability is set to $\langle \dot{\mu} \rangle$ in its cross section. That is, if the coil is parallel to x-axis, for instance, the permeability tensor is set to $\boldsymbol{\mu} = \mathrm{diag}[\mu_0\mu_{\mathrm{r}}, \langle \dot{\mu} \rangle, \langle \dot{\mu} \rangle]$. When there is no conductor except the coil, the field can be determined by solving the magnetostatic equation

$$\mathrm{rot}\boldsymbol{\nu}(\mathrm{rot}\boldsymbol{A}) - \boldsymbol{J} = 0 \tag{3}$$

where $\boldsymbol{A}$, $\boldsymbol{J}$ and $\boldsymbol{\nu}$ denote the vector potential, current density and reluctivity tensor $\boldsymbol{\nu}=\boldsymbol{\mu}^{-1}$, respectively. When there are additional conductors, the eddy current term is included in (3). Moreover, the circuit equations of the power transmission and receiving coils

$$\frac{R_k z J_0(z)}{2J_1(z)} I_k + \mathrm{j}\omega \Phi_k = V_k \tag{4}$$

are coupled with (3), where R_k, I_k, V_k, Φ_k (k=1,2) and ω denote the DC resistance, circuit current, input voltage, interlinkage flux and angular frequency, respectively. The first terms in (4) include the impedance coming from the skin effect.

B. Synthesis of equivalent circuit

We consider here a simple WPT device shown in Fig. 1 which is composed of an axisymmetric coil. In this case, (3) is reduced to two-dimensional equation including the scalar permeability $\langle \dot{\mu} \rangle$. The WPT is modeled by the equivalent circuit shown in Fig. 2, where the impedance is here represented by the Foster circuit [4]-[5] as follows:

$$Z(s)=\sum_{k=1}^{n}\frac{1}{\frac{1}{R_k}+\frac{1}{sL_k}} \tag{5}$$

where R_k, L_k and n denote resistance, inductance and number of the stage of the circuit, respectively. The circuit parameters $\boldsymbol{R} = [R_1, R_2, \ldots, R_n]$, $\boldsymbol{L} = [L_1, L_2, \ldots, L_n]$ in (5) are determined by solving the optimization problem defined by

$$f(\boldsymbol{R},\boldsymbol{L})=\sqrt{\sum_{i}^{M}\left|Z^{\mathrm{FEM}}(s_i)-Z(s_i,\boldsymbol{R},\boldsymbol{L})\right|^2}\rightarrow \min. \tag{6}$$

$$\text{s.t. } R_k, L_k \geq 0$$

where $Z^{\mathrm{FEM}}(s_i)$, $Z(s_i, \boldsymbol{R}, \boldsymbol{L})$ denote the impedance obtained by the homogenization-based FEM and equivalent circuit, and M denotes the number of sampling points for fitting [4]-[5]. After determining the impedance by solving (6), we convert the Foster circuit to the Cauer circuit shown in Fig. 3 to which we can give clearer physical interpretation; the primal resistance and inductance R_1 and L_1 correspond to the DC resistance and inductance to the flux generated by the external current, whereas R_k and L_k, k=2,3,… are relevant to the eddy current losses and fluxes generated by the eddy currents. By applying the Euclid's algorithm to the impedance function, we obtain

$$Z_i(s)-Z_{\mathrm{M}}(s)=R_{\mathrm{DC}i}+\cfrac{1}{\cfrac{1}{sL_{i1}}+\cfrac{1}{R_{i1}+\cfrac{1}{sL_{i2}+\cdots}}} \tag{7a}$$

$$Z_{\mathrm{M}}(s)=\cfrac{1}{\cfrac{1}{sL_{\mathrm{M1}}}+\cfrac{1}{R_{\mathrm{M1}}+\cfrac{1}{sL_{\mathrm{M2}}+\cdots}}} \tag{7b}$$

where $R_{\mathrm{DC}i}$, L_{i1} and L_{M1} denote the DC resistance, self-inductance and mutual inductance, respectively. From (7), we can immediately synthesize the equivalent circuit shown in Fig. 4.

III. RESULTS

The time response of the WPT shown in Fig. 1 is computed by the synthesized circuit shown in Fig. 4 and conventional equivalent circuit, shown in Fig. 5, in the latter of which eddy currents in the coil are not considered. In the analysis, the parameters are set as follows: a=0.15 mm, h=10 mm, D_{in}=60 mm, D_{out}=64.8 mm, R_{LOAD}=300 Ω. The time response is also measured. From the results in Fig. 6, we can see that the proposed circuit has smaller error compared with the conventional equivalent circuit.

In the full paper, we will describe the equivalent circuit synthesis in detail. Moreover, we discuss the three-dimensional model in which the WPT coils are tilted.

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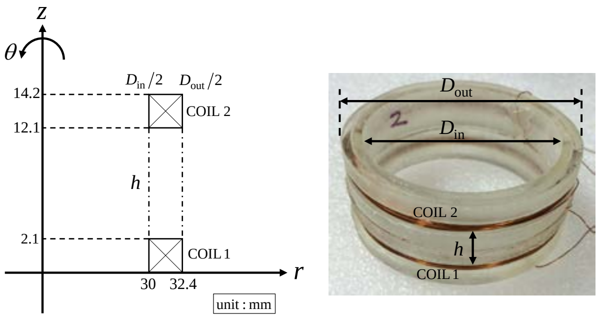

Fig. 1. Analysis model

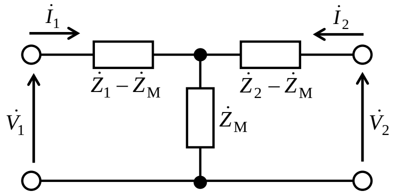

Fig. 2. T equivalent circuit

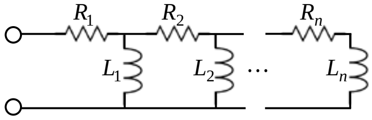

Fig. 3. Cauer circuit

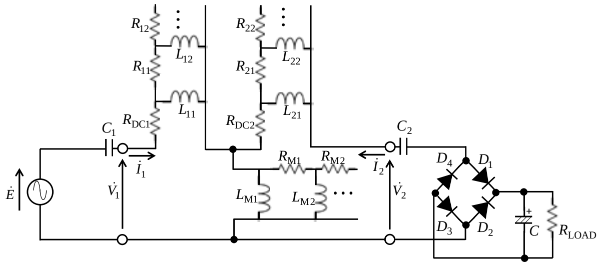

Fig. 4. Proposed equivalent circuit

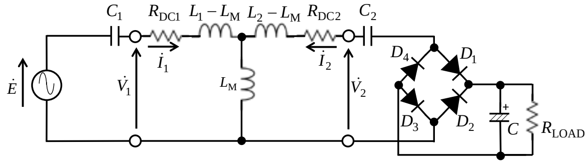

Fig. 5. Conventional equivalent circuit

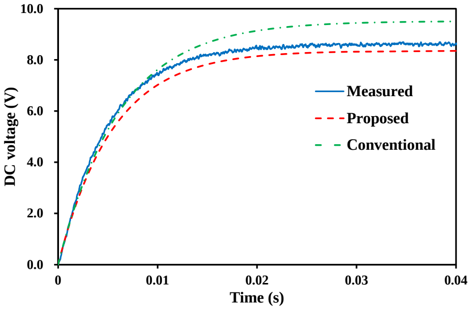

Fig. 6. Output DC voltages

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